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Concrete Members with FRP Bars under Transverse Thermal Loading

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ABSTRACT

The coefficient of transverse thermal expansion (CTE) of fiber reinforced polymers (FRP) bars can be substantially higher than that of concrete. Under temperature increase, this thermal incompatibility can result in tensile stresses that may produce cracking in concrete around the FRP bar. Nonlinear finite element (FE) analysis is employed to study this phenomenon in a concrete cylinder concentrically reinforced with an FRP bar. The FE analysis is first compared with a simplified elastic solution. Then, parametric studies of the main variables that are believed to influence the overall behavior of the problem at hand are presented. The studied variables include CTE and transverse modulus of elasticity of FRP bars, concrete strength, and concrete-cover-to-bar-diameter ratio. The effect of each parameter on the temperatures at which cracking starts to occur and at which a crack propagates to the outside surface of the concrete cover (splitting crack) is evaluated. Based on the analysis of the finite element results, it is concluded that FE analysis shows conservative predictions of transverse thermal loading of studied concrete cylinders concentrically reinforces with FRP bars.

Keywords

Concrete cover; cracking; fiber reinforced polymers; finite element analysis; temperature; thermal stresses.

1. INTRODUCTION

Unlike conventional steel reinforcement, the coefficient of thermal expansion of fiber reinforced polymer (FRP) bars differs considerably from that of concrete. Thermal properties of FRP bars vary widely from one product to another depending on the type of fiber and resin matrix as well as on the fiber volumetric ratio. In addition, FRP bars are anistropic materials; i.e., their mechanical and thermal properties in the longitudinal direction are different from those in the transverse direction. The modulus of elasticity and the coefficient of thermal expansion (CTE) in the transverse direction are, respectively, lower and much higher than their counterparts in the longitudinal direction. This is because, in the longitudinal direction, the properties of

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FRP bars are strongly dependent on the fibers because of their orientation, while the resin matrix plays a more dominant role in the transverse direction. Carbon fiber reinforced polymers (CFRP), for example, have longitudinal CTE close to zero [8] but its transverse CTE can be 4 – 7 times higher than the corresponding value for concrete (Rahman et al. 1995[12]). On the other hand, glass fiber reinforced polymers (GFRP), while having a longitudinal CTE similar to that of concrete, their transverse CTE could be 5 – 8 times greater than concrete [6]; [11].

The problem of thermal incompatibility between FRP bars and the surrounding concrete in the longitudinal direction is relatively simple. Elbadry et al. [7] presented a closed-form solution for the stresses in concrete and FRP bars resulting from thermal loading (uniform temperature rise or thermal gradient) of a concrete prism reinforced either concentrically or eccentrically with a number of FRP layers. On the other hand, the problem in the transverse direction is rather complex and a closed-form solution is not always possible. Due to the relatively greater transverse CTE of an FRP bar, a temperature rise causes the bar to exert a compressive radial force on the surrounding concrete. This radial force causes splitting stresses in the concrete that may lead to cracking if the tensile strength of the concrete is exceeded and as a result potential damage to the bond between the FRP bar and concrete may occur. Moreover, with a decrease in temperature, the FRP bar may shrink away from the concrete leading to partial separation and complete loss of bond.

The interaction between FRP bars and concrete in the transverse direction under uniform thermal loading has been a subject of considerable research during the past decade. Rahman et al. [12] used the theory of elasticity (explained in detail in the following section) to calculate the temperature at which concrete starts to crack around the FRP bar. An axisymmetric problem was assumed for a case of concrete cylinder concentrically reinforced with FRP bar. However, this elastic solution cannot be extended to predict the temperature at splitting cracking (cracks that propagate through the cover to reach the outside surface of the concrete) due to the nonlinear concrete behavior after cracking. Matthys et al. [11] studied the influence of transverse thermal expansion of aramid prestressing elements near the anchorage zones on the critical concrete cover to prevent splitting cracks. Parametric studies of different variables, which were believed to be influential, were conducted using nonlinear 2D finite element analysis, and corresponding critical values for concrete cover for each case were suggested by the authors. Aiello [3] presented an analytical model to evaluate the critical concrete cover and the minimum amount of transverse reinforcement to avoid cover spalling caused by splitting cracks. The proposed model predicts results in good agreement with experimental results only for low cover-to-bar-diameter ratios c/d_b, but yields unsatisfactory results with c/d_b higher ratios. Nevertheless, the mathematical expressions used in the model are rather lengthy and complicated, which may inhibit its use by practicing engineers.

Nonlinear finite element analysis is adopted in this paper to conduct parametric study of different variables that affect the behavior of FRP bars in concrete cylinders under transverse thermal loading. The studied variables include CTE and modulus of elasticity of FRP bars, concrete strength, and concrete-cover-to-bar- diameter ratio. The temperatures at which cracking starts to develop and at which a crack reaches the surface of the member are determined.

2. SIMPLIFIED ELASTIC ANALYSIS

Timoshenko and Goodier [13] presented an elastic solution to calculate the tangential stresses in a hollow cylinder subjected to internal radial pressure p assuming plane stress condition. Rahman et al. [12]; Masmoudi et al. [10] and Abdalla [1] adopted the same approach but for an FRP bar embedded in a concrete cylinder under the effect of increasing temperature. The relatively greater transverse expansion of the FRP bar produces radial pressure on concrete that can be calculated as follows:

$$p = \frac{\left(\alpha_{\mathrm{fT}} - \alpha_{\mathrm{c}}\right) T n_{\mathrm{fT}} E_{\mathrm{c}}}{\left[n_{\mathrm{fT}}\left(\beta + \nu_{\mathrm{c}}\right) + \left(1 - \nu_{\mathrm{fT}}\right)\right]} \quad [1]$$

where $n_{ft} = E_{fT}/E_c$ is the modular ratio in the transverse direction, with E_{fT} being the transverse modulus of elasticity of the FRP bar; υ_{fT} = in-plane Poisson's ratio of the FRP bar; and υ_c = Poisson's ratio of concrete. The coefficient β depends on the FRP bar diameter d_b and the concrete cover c:

$$\beta = \left(b^2 + a^2\right) \Big/ \left(b^2 - a^2\right) \quad [2]$$

where $b=c+d_b/2$ and $a=d_b/2$ are the radii of the concrete cylinder and the FRP bar, respectively. The tangential stress σ_t induced in the concrete at any radius r (Fig. 1) by the radial pressure p is given by:

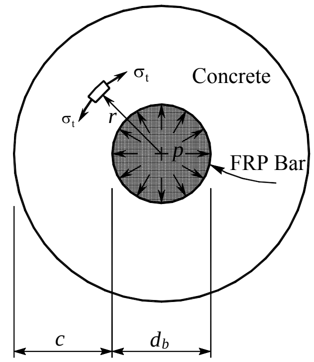


Fig. 1: Radial stress in concrete due to transverse thermal expansion of FRP bar

$$\sigma_{\mathrm{t}} = \frac{a^2\left(b^2 + r^2\right)}{r^2\left(b^2 - a^2\right)} p \quad [3]$$

with the maximum stress occurring at the interface between the FRP bar and the concrete (i.e., at $r = a$). The temperature at first cracking can be evaluated from eq. [3] and noting eq. [1] by setting σ_t equal to the tensile strength of concrete and solving for p.

3. NONLINEAR FINITE ELEMENT ANALYSIS

The elastic solution presented above is useful, but limited to axisymmetric problems and serve only to indicate the temperature increase at which cracking starts to take place. The extent of this cracking and the distribution of stresses in the concrete cover around the bar cannot be predicted by the simplified model presented above. Nonlinear FE analyses are employed using software code ADINA to: (1) verify it against the elastic solution; and (2) further study the problem after cracking starts to occur.

A concrete cylinder concentrically reinforced with an FRP bar will be analyzed under uniform thermal loading. By selecting a section in the cylinder away from its edges to exclude any end conditions effects, the problem can be solved as a plane-stress problem. Furthermore, as far as transverse direction is the only direction concerned only here, the mechanical and thermal properties of the FRP bar can be considered isotropic. The geometry of the problem represents a critical case of a corner bar in a concrete beam where the initiation and extent of cracking is governed by the least cover dimension.

Finite element model

The geometry of the finite-element mesh is shown in Fig. 2, where the location of the FRP bar is shaded in the figure. Due to symmetry of the problem in both directions, only one-quarter of the cross-section is analyzed and appropriate displacement components are restrained along the symmetry lines of the model. Eight-node plane-stress elements with full integration are used for modeling both the concrete and the FRP bar.

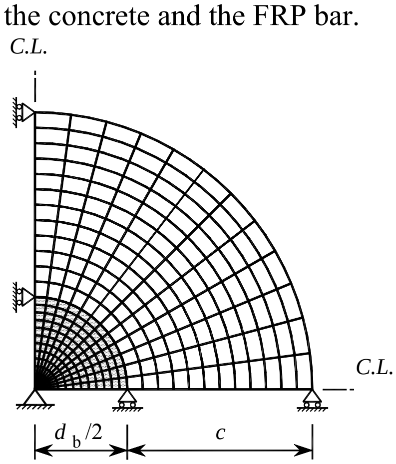


Fig. 2: Typical finite element mesh (support conditions are not shown at all points)

The concrete model available in ADINA is adopted in the present analysis. The model is based on a hypoelastic nonlinear stress-strain formulation with failure envelopes that model both compression and tensile failures. The concept of equivalent uniaxial stress-strain relation is used to take into account the behavior of concrete under biaxial and triaxial state of stresses. The model accounts for the tension stiffening of concrete and for the gradual stress release in the strain softening region. Further description and detailed constitutive relations of the concrete material model can be found elsewhere (ADINA [2]; Bathe and Sundberg [5]). The composite FRP material, on the other hand, is modeled using isotropic thermo-elastic material model, where the mechanical properties (Elasticity modulus and Poisson's ratio) are assumed temperature independent.

The thermal loading is applied on the model in small increments (1 - 2°C) of uniform temperature increase. The cracking pattern in concrete around the bar is monitored to record the temperatures at which cracking starts to occur as well as when it penetrates the concrete cover to reach the outside surface.

Material properties

The variables investigated in the current study include concrete compressive strength $f_{c'}$, transverse coefficient of thermal expansion CTE and modulus of elasticity of FRP E_{fT} as well as concrete-cover-to-bar-diameter-ratio c/d_b. The range of variation for each parameter is listed in Table 1. The concrete compressive strength is varied to reflect common values used in practice. Different values for CTE and E_{fT} are considered in order to cover the properties of FRP bars reported by researchers (Rahman et al. [12]; Gentry and Hudak [9]). Previous research has indicated that Poisson's ratios for both concrete and FRP materials vary within a narrow margin; therefore, they are assumed constant throughout the analyses and taken as 0.18 and 0.35 for concrete and FRP, respectively. The coefficient of thermal expansion of concrete is taken as 10×10^{-6}/°C and the concrete tensile strength is assumed to be 10% of its compressive strength. The used ADINA concrete model requires the initial tangent modulus E_{ci} to be defined in the input data. The following expression for E_{ci} is used (SI units):

$$E_{ci} = 5500\sqrt{f'_c} \qquad [4]$$

Table 1: Variables investigated in the parametric studies of the FE analysis

Variable	Range of variation
Concrete strength, f'_c	30 – 60 MPa
Transverse CTE of FRP	20 – 60 × 10^{-6}/°C
Cover-to-bar-diameter ratio, c/d_b	1 – 6
Transverse modulus of elasticity of FRP, E_{fT}	3000 – 5000 MPa

Comparison between elastic and finite element solutions

Figure 3 shows a comparison between the temperatures at first crack obtained from the elastic solution of eq. [3] and from FE analysis for different values of c/d_b ratios. Other parameters such as $f_{c'}$ and CTE are kept unchanged. The initiation of cracking was recognized in the FE model when the concrete tensile strength was exceeded at the row of integration points of the concrete elements nearest the concrete/bar interface. As shown in the figure, the elastic and FE solutions compare very well at ratios of c/d_b less than 2. At ratios greater than 2, the FE solution starts to predict temperatures higher than the elastic solution. This is because the first row of integration points nearest the interface is shifted away as the aspect ratio of the concrete elements increases. The difference between the FE and elastic solutions is reduced (Fig. 3) when using refined FE mesh near the concrete/bar interface.

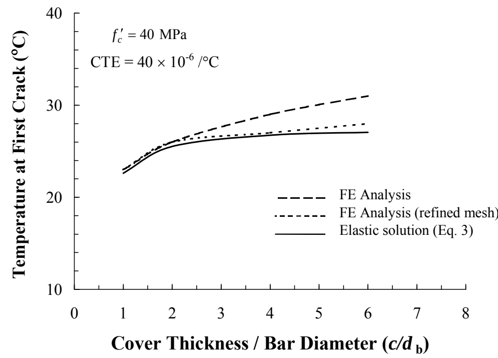


Fig. 3: Comparison between elastic and FE solutions for temperature at initiation of cracking

Crack pattern and distribution of stresses

After verifying the FE solution against the elastic solution, the problem is studied further after cracking has occurred. The crack patterns at different temperatures are depicted in Fig. 4. The variables assumed in this case were: fc'= 40 MPa; CTE = 40×10^{-6}/°C; and c/d_b = 2. The cracking initiated at 26°C at the concrete/bar interface (Fig. 4(a)). With further increase in temperature, one of the cracks around the FRP bar started to propagate through the concrete cover (Fig. 4(b)) until it reached the outside surface, as shown in Fig. 4(c). This sequence of cracking was typical for all cases considered in the parametric study. The distribution of principal stresses associated with the crack patterns of Fig. 4(a) and (b) is shown in Fig. 5(a) and (b), respectively.

The distribution of tensile (splitting) stresses in concrete cover at different temperatures along a potential crack line is illustrated in Fig. 6 for the same case studied in Figs. 4 and 5. The stress started to build up away from the concrete/bar interface (as distance x/d_b increases) until it reached the concrete tensile strength (4 MPa in this case). As the concrete

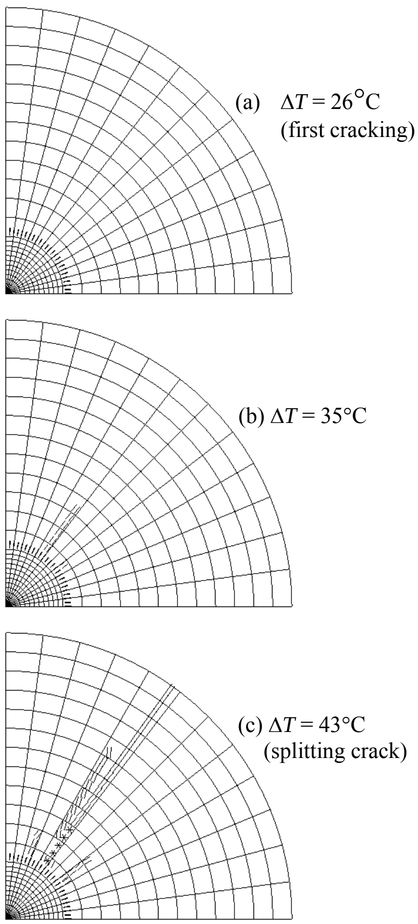


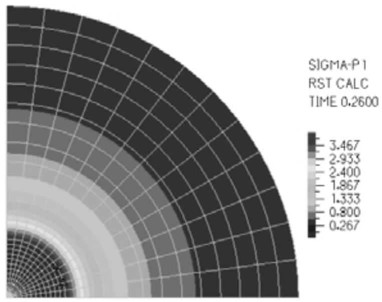
Fig. 4: Propagation of cracking (f_c' = 40 MPa; CTE = 40 × 10^{-6}/°C; c/d_b = 2

tensile strength was reached, the stress was released and with further increase in temperature started to build up again at a further point along the potential crack line until it reaches the outside surface.

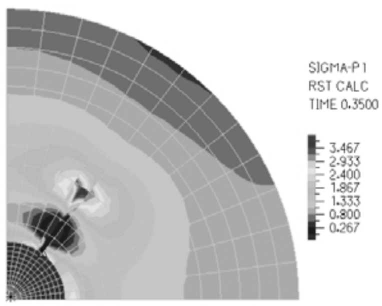
4. PARAMETRIC STUDY

As mentioned above, the main variables that are recognized as having the most influential effect on the behavior of FRP bars in concrete under transverse thermal loading are: the concrete strength, $f_{c'}$; the transverse coefficient of thermal expansion of FRP, CTE; the modulus of elasticity of FRP in the transverse direction, E_{fT}; and the concrete cover-to-bar-diameter ratio, c/d_b . Two main events are identified in the analysis: the temperature at first crack and the temperature at splitting crack (or through crack). As cracking starts to take place at the interface of the FRP bar and the surrounding concrete, bond between the two materials starts to deteriorate and consequently the splitting tensile strength of concrete is reduced. Elbadry et al. [7] reported a significant reduction in the splitting tensile strength of concrete cylinders reinforced with either GFRP or AFRP bars at high temperatures compared to identical tests conducted at room temperature. In almost all of tested cylinders, no splitting cracks were observed. Therefore, in applications where the tensile strength of concrete is critical in design, the temperature at first crack gives an indication as to when the tensile strength of concrete starts to deteriorate. On the other hand, a splitting crack causes complete loss of the bond between the FRP bar and concrete and may eventually lead to spalling of the concrete cover. It is believed that in most practical applications the temperature at splitting crack will govern the choice of the concrete cover.

The effect of concrete strength on temperature at first crack and on temperature at splitting crack is shown in Figs. 7 and 8, respectively. In both figures, the transverse CTE of FRP is kept constant at 40×10^{-6}/°C.



(a) ΔT = 26°C (first cracking)



(b) ΔT = 35°C

Fig. 5: Contour maps of principal stresses (f_c' = 40 MPa; CTE = 40 × 10^{-6}/°C; c/d_b = 2)

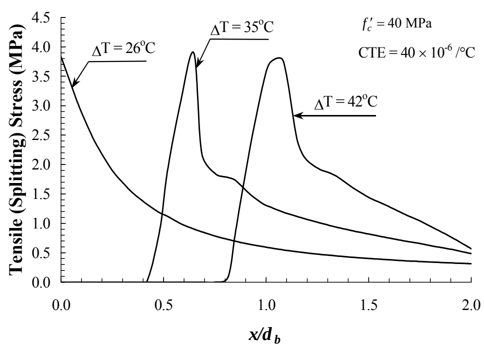


Fig. 6: Tensile stress distribution along potential crack line (f_c' = 40 MPa; CTE = 40 × 10^{-6}/°C; $c/d_b = 2$)

The concrete strength affects the results indirectly in two ways. Increasing the concrete strength translates into higher modulus of elasticity and higher tensile strength. Higher modulus of elasticity of concrete means more restraint to the transverse expansion of the FRP bar and hence cracking can take place at lower temperatures. On the other hand, higher tensile strength of concrete delays the occurrence as well as the propagation of cracking through the concrete cover. It is apparent from both figures that increasing the concrete strength increases both the temperature at first and splitting crack. This indicates that the effect of the tensile strength of concrete plays a more dominant role on overall performance than the modulus of elasticity.

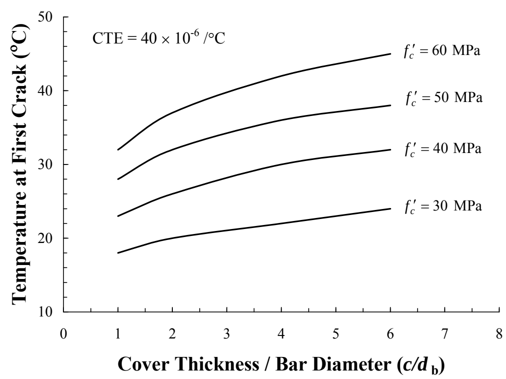


Fig. 7: Effect of concrete strength on temperature at initiation of cracking

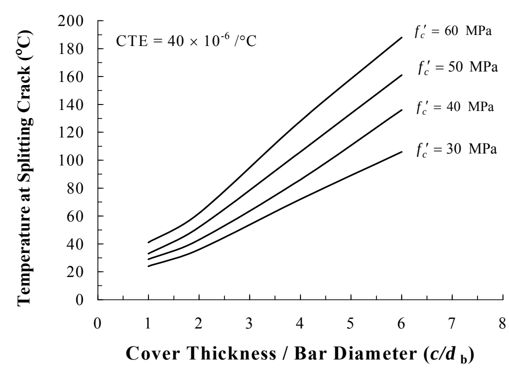


Fig. 8: Effect of concrete strength on temperature at splitting crack

It also appears from Figs. 7 and 8 that the temperature at first and splitting crack increases almost linearly with the concrete strength. Figure 7 shows that increasing the c/d_b ratio from 1 to 2 increase the temperature at first crack substantially. Increasing c/d_b ratio above 2 results in a modest increase of temperature, especially for lower concrete strengths. Figure 8 shows a different trend: increasing c/d_b ratio increases the temperature at splitting crack exponentially.

Figures 9 and 10 illustrates the effect of the transverse CTE of the FRP bar on the temperature at first and splitting cracking, respectively. The concrete strength in both cases is kept unchanged at 40 MPa. As expected, higher CTE values results in early cracking of the concrete since, for a given temperature increase, FRP bars with higher CTE values exerts greater radial pressure on the surrounding concrete and hence creates higher splitting stresses in the surrounding concrete. It can be observed from both figures that higher CTE values result in a less than proportional decrease in temperatures at cracking. For instance, in a concrete cylinder with c/d_b ratio of 2, the temperature at splitting crack is reduced by 23°C (from 66°C to 43°C) for an increase in CTE from 30×10^{-6} to 40×10^{-6}/°C, while an additional 10×10^{-6}/°C increase in CTE reduces the temperature at splitting crack by only 10°C (from 43°C to 33°C).

Similar to the trend observed in Figs. 7 and 8, the effect of concrete cover is more pronounced in the case of temperature at splitting crack than in the case of temperature at first crack. This is because at first cracking the cover thickness plays a minor role in confining the FRP bar from expanding laterally, whereas after cracking starts to occur, the thicker concrete cover delays the propagation of a potential crack to the surface of the concrete cylinder. It should be mentioned that in Figs. 4 through 10 the transverse modulus of elasticity of FRP bar, E_{fT}, is assumed 3800 MPa. This value was reported by Rahman et al. [12] and Aiello et al. [4] for two types of GFRP bars.

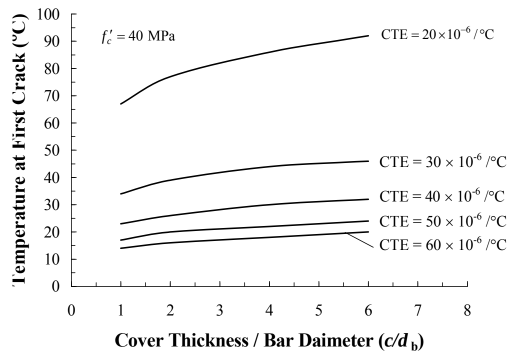


Fig. 9: Effect of coefficient of transverse thermal expansion (CTE) of FRP on temperature at initiation of cracking

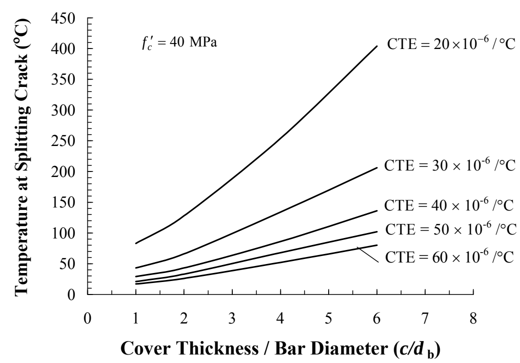


Fig. 10: Effect of coefficient of transverse thermal expansion (CTE) of FRP on temperature at splitting crack

The effect of the transverse modulus of elasticity of FRP bar, E_{fT}, can be evaluated from Figs. 11 and 12 for temperature at first and splitting crack, respectively. From a survey of the limited data available in the literature about the values of E_{fT}, upper and lower limits of E_{fT} were found to be 3000 and 5000 MPa, respectively. The trend in Figs. 11 and 12 is very clear. An increase in E_{fT} accelerates the initiation and propagation of cracking.

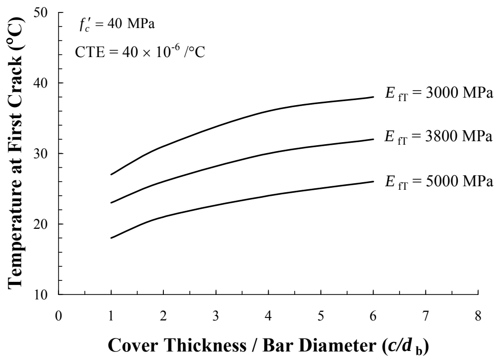


Fig. 11: Effect of transverse modulus of elasticity of FRP on temperature at initiation of Cracking

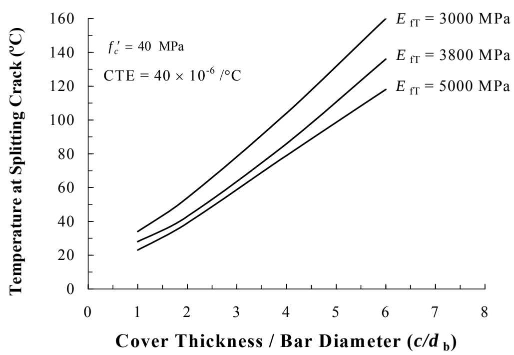


Fig. 12: Effect of transverse modulus of elasticity of FRP on temperature at splitting crack

For a given maximum temperature that a structural member is expected to subjected to during its life span, the design engineer can use any of the curves in Figs. 7 through 12 or interpolate between them to find the suitable c/d_b ratio to avert the possibility of first or splitting crack. It should be mentioned that the analysis in this paper represents a lower bound (conservative) solution to the real problem in practice. This is due to the fact that a rise of temperature normally takes some time to develop and stabilize. During this period, even few hours, creep of concrete alleviates the stresses induced due to temperature. In addition, presence of transverse reinforcement around the bar (as in the case of concrete beams) can delay the propagation of cracking to the outside surface of the concrete member.

5. CONCLUSIONS

Based on the analytical and experimental investigations reported in this paper, the following conclusions can be made:

1. Due to the large difference in the coefficients of thermal expansion of concrete and FRP bars in the transverse direction, any temperature increase induces tensile stresses in the surrounding concrete. If the tensile strength of concrete is exceeded, cracking can initiate at the bar/concrete interface and subsequently propagates through the concrete cover to the outside surface.
2. The finite element analysis shows conservative predictions of the temperature at first cracking compared to the elastic solution suggested by Rahman et al. [12].
3. Increasing the concrete cover-to-bar-diameter ratio and/or the concrete strength delays the initiation of cracking and its propagation through the concrete cover. On the other hand, higher coefficient of thermal expansion and transverse modulus of elasticity of the FRP bar cause the initiation and propagation of cracking to take place at lower temperatures.
4. Increasing the concrete cover is more effective in delaying the propagation of cracking to the surface rather than in preventing the initiation of cracking.

6. ACKNOWLEDGEMENTS

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ملخص البحث

معامل التمدد الحرارى العرضى للأسياخ المصنعة من الألياف المقواة بالبوليمرات (FRP) من الممكن

أن يزيد زيادة مرتفعة عن مثيلة للخرسانة. تحت تأثير أرتفاع درجات الحرارة يتسبب عدم التجانس فى معامل التمدد الحرارى فى حدوث أجهادات شد و التى قد تؤدى إلى تشرخات بالخرسانة حول السيخ. فى هذا البحث تم أستخدام طريقة العناصر المحددة بغرض دراسة هذة الظاهرة وذلك على أسطوانات الخرسانة المسلحة مركزياً بأستخدام أسياخ FRP. فى بادئ الأمر تم مقارنة نتائج التحليل بواسطة العناصر المحددةً مع الحل المرن. يشتمل هذا البحث أيضاً على دراسة بارامترية بتغيير المتغيرات الرئيسية والتى من المتوقع أن يكون لها تأثير على السلوك العام للأسطوانات. المتغيرات التى تم دراستها تشمل معامل التمدد العرضى و معامل المرونة العرضى لأسياخ FRP و مقاومة الخرسانة بالأضافة إلى نسبة الغطاء الخرسانى إلى قطر السيخ. تم تقييم تأثير كل من تلك المتغيرات على درجة الحرارة التى يبدأ عندها حدوث شروخ بالخرسانة بالأضافة إلى درجة الحرارة التى يصل عندها الشرخ إلى السطح الخارجى للغطاء الخرسانى. بناء على دراسة نتائج التحليل بأستخدام طريقة العناصر المحددة فقد تم أستنتاج أن هذة الطريقة تعطى قيم محافظة للحمل الحرارى العرضى للأسطوانات الخرسانية المسلحة مركزياً بأستخدام أسياخ FRP والتى تمت دراستها.